Pion as a Longitudinal Axial-Vector Meson $q\bar{q}$ Bound State

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The success of the Adler-Bell-Jackiw(ABJ) chiral anomaly prediction for $\pi^0 \to \gamma\gamma$ decay rate shows that non-anomaly terms would make a negligible contribution to the decay rate, in agreement with the Sutherland-Veltman theorem. Thus the conventional $q\bar{q}$ bound-state description of the pion could not be valid since it would produce a $\pi^0 \to \gamma\gamma$ decay amplitude not suppressed in the soft pion limit, in contradiction with the Sutherland-Veltman theorem. Therefore, if the pion is to be treated as a $q\bar{q}$ bound state, this bound state would be a longitudinal axial-vector meson. In this paper, we consider the pion to be a longitudinal axial-vector meson $q\bar{q}$ state with derivative coupling for the pion-$q\bar{q}$ Bethe-Salpeter(BS) amplitude. We shall show that, the longitudinal axial-vector meson solution for the pion $q\bar{q}$ Bethe-Salpeter wave function could produce a suppressed $\pi^0 \to \gamma\gamma$ decay amplitude in the soft pion limit, in agreement with the Sutherland-Veltman theorem.

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Chiral symmetry, as known from the success of the Goldberger-Treiman relation for the pion-nucleon coupling constant obtained from the PCAC hypothesis, is a good symmetry of strong interactions. The spontaneous breakdown of the $SU(2) \times SU(2)$ chiral symmetry generates a massless Nambu-Goldstone boson which then acquires a small mass through a chiral symmetry breaking quark mass term. PCAC and Adler-Bell-Jackiw chiral anomaly [1–3] then produce the $\pi^0 \to \gamma\gamma$ decay rate in good agreement with experiment. On the other hand, in a conventional bound-state model, a neutral pseudoscalar $q\bar{q}$ 0^{-+} state, like the η_c meson, is usually massive and could decay into two photons like the two-photon decays of positronium and heavy quarkonium. Being massive, they cannot be identified with the neutral pseudoscalar meson of the ground state $SU(3)$ octet like π^0 and η meson, the Nambu-Goldstone bosons of the $SU(3) \times SU(3)$ chiral symmetry. In the traditional non-relativistic and relativistic bound-state calculations, one could compute the $\pi^0 \to \gamma\gamma$ decay rate using the physical pion mass and obtains some agreement with experiment [4–7], but this particle could not be the pion, since the two-photon decay amplitude for this pseudoscalar $q\bar{q}$ state is not suppressed in the soft pion limit according to the Sutherland-Veltman theorem [8, 9]. The pion could however be in a longitudinal axial-vector meson $q\bar{q}$ state, if this state could produce a suppressed $\pi^0 \to \gamma\gamma$ decay amplitude in the soft pion limit so that the

agreement with experiment for the ABJ anomaly prediction of the π^0 two-photon decay rates is preserved. In this paper, we shall show that, with the longitudinal axial-vector meson pion BS wave function, the $\pi^0 \to \gamma\gamma$ decay amplitude would be suppressed in the soft pion limit, in agreement with the Sutherland-Veltman theorem. Since the basis of our analysis is the Sutherland-Veltman theorem, for convenience, we reproduce this theorem here. Writing the $\pi^0 \to \gamma\gamma$ amplitude in the original notation [8], we have:

$$g\,\epsilon_{\alpha\beta\gamma\delta}\epsilon_{1\alpha}\epsilon_{2\beta}k_{1\gamma}k_{2\delta} = \epsilon_{1\alpha}\epsilon_{2\beta}\int <0|\mathrm{T}[j_{1\alpha}(x)\,j_{2\beta}(0)|\pi^0_q> \exp{(-i\,k_1\cdot x)}d^4x. \tag{1}$$

Using PCAC with

$$\partial_\mu A^\mu = f_\pi m_\pi^2 \varphi_\pi, \tag{2}$$

one finds:

$$\begin{aligned}&\frac{(q^2-m_\pi^2)}{f_\pi m_\pi^2}\,\epsilon_{1\alpha}\epsilon_{2\beta}\int <0|\mathrm{T}[j_{1\alpha}(x)\,j_{2\beta}(0)\partial_\mu j^5_\mu(z)|0>\ \exp{(-i\,k_1\cdot x+i\,q\cdot z)}d^4x\,d^4z\\ &=\frac{(q^2-m_\pi^2)}{f_\pi m_\pi^2}\,\epsilon_{1\alpha}\epsilon_{2\beta}\,q_\mu\int <0|\mathrm{T}[j_{1\alpha}(x)\,j_{2\beta}(0)j^5_\mu(z)|0>\ \exp{(-i\,k_1\cdot x+i\,q\cdot z)}d^4x\,d^4z.\end{aligned} \tag{3}$$

Since gauge invariance requires that

$$\int <0|\mathrm{T}[j_{1\alpha}(x)\,j_{2\beta}(0)j^5_\mu(z)|0>\ \exp{(-i\,k_1\cdot x+i\,q\cdot z)}d^4x\,d^4z \propto \epsilon_{\alpha\beta\nu\sigma}k_{1\nu}k_{2\sigma}q_\mu. \tag{4}$$

for $q^2=0$ (q being the pion momentum), $g\to 0$ in the soft pion limit, the amplitude $\pi^0\to\gamma\gamma$ is $O(q^2)$ and becomes suppressed as $q^2\to 0$. This theorem is now evaded by the ABJ anomaly which gives us the well-known chiral anomaly prediction for $\pi^0\to\gamma\gamma$ decay : $g=-(\alpha/4\pi)\,/f_\pi$ [10].

We have seen that without the ABJ anomaly, the $\pi^0\to\gamma\gamma$ decay would be suppressed. In any model calculation, for example, in non-relativistic or relativistic calculation, without PCAC and chiral symmetry, the Sutherland-Veltman theorem does not apply and the two-photon decay is not suppressed in the soft pion limit as found in existing bound-state calculations of quarkonium two-photon decays [4–7]. It follows that the pion could not be described by the usual $^1S_0$ state momentum-independent $q\bar{q}$ bound-state wave function. Since many properties of hadrons, and in particular, the light mesons and quarkonium systems, are well described by the $q\bar{q}$ bound-state picture, the problem is how to reconcile this bound-state picture with the Nambu-Goldstone boson character of the pion. The solution of the problem could be found easily by looking at the solution of the relativistic bound-state Bethe-Salpeter(BS) equation [11] for a $q\bar{q}$ system. For a pseudoscalar meson, there are two possible solutions. The pseudoscalar solution with the momentum-independent wave function of the form $P\gamma_5$ and the longitudinal axial-vector momentum-dependent

$\not{p}\gamma_5 A$ solutions. As mentioned above, the pseudoscalar solution would be in contradiction with the Sutherland-Veltman theorem and therefore could not be the correct pion $q\bar{q}$ bound-state wave function. The longitudinal axial-vector solution would be acceptable. In fact, if the pion is a longitudinal axial-vector meson $q\bar{q}$ bound state, the $\pi^0 \to \gamma\gamma$ amplitude computed with this wave function, as shown below, would be similar to the free quark triangle graph contribution to the two-photon matrix element of the axial-vector current divergence $< 0|\partial_\mu A_\mu(0)|\gamma\gamma >$ and therefore vanishes in the massless quark limit and thus does not contribute to the $\pi^0 \to \gamma\gamma$ decay. In the following we present a computation of the $\pi^0 \to \gamma\gamma$ amplitude using the longitudinal axial-vector meson as the pion BS wave function [12]:

$$\psi(p,q) = \gamma_5\psi_0 + \gamma_5 \not{p}\psi_1 + \gamma_5 \not{q} p\cdot q\psi_2 + \gamma_5[\not{q}, \not{p}]\psi_3. \tag{5}$$

where p and q is the pion and relative momentum of the $q\bar{q}$ system, with the quark and anti-quark momwentum $q_1 = q + p/2$, $q_2 = q - p/2$ and $\psi_i, i = 0, ..., 3$ are the scalar functions of p and q. The first term ψ_0 in Eq. (5) is the momentum-independent wave function, as mentioned above, produce a $\pi^0 \to \gamma\gamma$ decay in the soft pion limit and is dropped here. The third term ψ_2 which is $O(p\cdot q)$ could give a contribution $O(p)$ in the soft pion limit and need not to be considered here. The last term ψ_3, does not make a contribution to $\pi^0 \to \gamma\gamma$ decay by the triangle graph. This leaves us with the ψ_1 term as the longitudinal contribution to the $\pi^0 \to \gamma\gamma$ decay. The BS equation [12] for $\psi(p,q)$ with the gluon propagator $G_{\mu\nu}(k-q)$ reads:

$$(\not{q}+ \not{p}/2)\psi(p,q)(\not{q}- \not{p}/2) = -i\int \frac{d^4q'}{(2\pi)^4}\gamma_\mu\psi(p,q')\gamma_\nu G_{\mu\nu}(q'-q). \tag{6}$$

Since, by definition, the BS vertex function $\Gamma(p,q)$ is the BS wave function with the free quark propagator removed [13, 14], Eq. (6) can be used to express $\Gamma(p,q)$ in terms of the BS wave function $\psi(p,q)$. We have:

$$\Gamma(p,q) = -i\int \frac{d^4q'}{(2\pi)^4}\gamma_\mu\psi(p,q')\gamma_\nu G_{\mu\nu}(q'-q). \tag{7}$$

In the following, as our purpose is to obtain the soft pion limit for $\pi^0 \to \gamma\gamma$ decay, we consider only the longitudinal solution for the BS wave function $\gamma_5 \not{p}\,\psi_1$ given in Eq. (5), and for simplicity, we use the gluon propagator in the Feynman gauge with $G\mu\nu(q'-q) = -g_{\mu\nu}/(q'-q)^2$. The $\pi^0 \to \gamma\gamma$ decay amplitude is given by the quark loop triangle graph similar to the ABJ chiral anomaly triangle graph, except that the point-like axial-vector current vertex $\gamma_\mu\gamma_5$ is replaced by the BS longitudinal axial-vector meson wave function $\psi(p,q') = \gamma_5 \not{p}\,\psi_1(p,q')$, and the factor

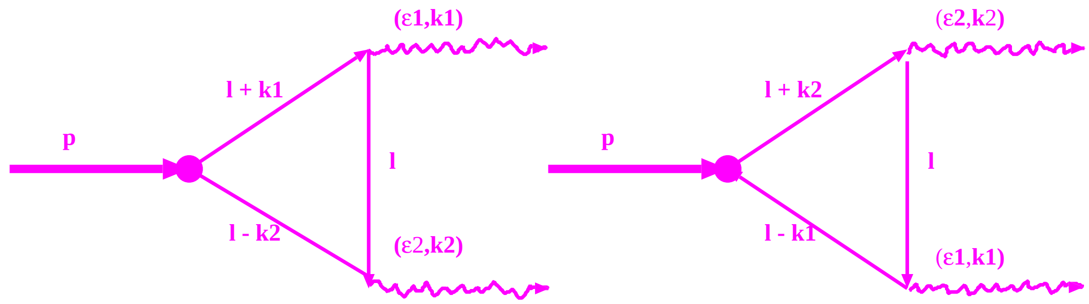

FIG. 1: Quark loop triangle graphs with BS longitudinal axial-vector meson wave function for $\pi^0 \to \gamma\gamma$ decay

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$1/(q'-q)^2$ from the gluon propagator which makes the integration over q convergent and could be carried out by the usual change of variable, assuming the integral over q' convergent. Similar to the calculation of Ref. [1], the $\pi^0 \to \gamma\gamma$ decay amplitude with the BS vertex function $\Gamma(p,q)$ shown in Fig. 1, after a change of variable $l = q + p/2$, with l one of the quark momentum in the triangle loop and $\Gamma(p,q) = \Gamma(p,l)$ and putting $m = 0$, is given by:

$$M = -ie^2 \int \frac{d^4l}{(2\pi)^4} \mathrm{Tr}\Big(\not\epsilon_2 \frac{1}{(\not l - \not k_2)} \not p \gamma_5 \frac{1}{(\not l + \not k_1)} \not\epsilon_1 \frac{1}{\not l} \Big)\, \mathrm{J(p,l)} + (\epsilon_1, \mathrm{k}_1 \to \epsilon_2, \mathrm{k}_2 \text{ terms}) \tag{8}$$

with the scalar part of the BS vertex function $\Gamma(p,l)$ given by:

$$J(p,l) = -2 \int \frac{d^4l'}{(2\pi)^4} \frac{\psi_1(p,l')}{(l'-l)^2}. \tag{9}$$

Using the identity [1],

$$\frac{1}{(\not l - \not k_2)} \not p \gamma_5 \frac{1}{(\not l + \not k_1)} = \frac{1}{(\not l - \not k_2)}\gamma_5 + \gamma_5 \frac{1}{(\not l + \not k_1)}, \quad p = k_1 + k_2 \tag{10}$$

The Dirac γ term (the Trace term), is then split into two contributions. The contributions from the 1st and 2nd diagrams in Fig. 1 are respectively then:

$$T_1 = \frac{\not\epsilon_2 (\not l - \not k_2)\, \not\epsilon_1\, \not l \gamma_5}{(l-k_2)^2 l^2} - \frac{\not\epsilon_2 (\not l + \not k_1)\, \not\epsilon_1\, \not l \gamma_5}{(l+k_1)^2 l^2} \tag{11}$$

$$T_2 = \frac{\not\epsilon_1 (\not l - \not k_1)\, \not\epsilon_2\, \not l \gamma_5}{(l-k_1)^2 l^2} - \frac{\not\epsilon_1 (\not l + \not k_2)\, \not\epsilon_2\, \not l \gamma_5}{(l+k_2)^2 l^2} \tag{12}$$

We see that, provided that the integral over l converges, the k_2-terms in Eq. (11) and Eq. (12) would cancel after integration over l by a change of variable $l - k_2 \to l$ and $l \to l + k_2$ in the k_2-terms of Eq. (11) and similarly for the k_1-terms with a change of variable $l - k_1 \to l$ and $l \to l + k_1$ in Eq. (12). This is not the case with point-like axial-vector current in the triangle graph since the shift of the integration variable $l - k_2 \to l$ in Eq. (11), or $l - k_1 \to l$ in Eq. (12), would induce an

anomaly term [15]. This is the well-known anomaly terms for the divergence of the axial-vector current[1]. In our bound-state calculation, the point-like axial-vector current is replaced by the longitudinal axial-vector meson BS vertex function and the $1/l^2$ behavior of the gluon propagator at large l^2 would make the integrals over l convergent for k_1 and k_2 terms in the two diagrams. Taking the trace, the total contribution to $\pi^0 \to \gamma\gamma$ decay amplitude is then given by:

$$M = -i\, e^2 \int \frac{d^4 l}{(2\pi)^4} \Big(-\frac{4i\epsilon(\epsilon_1, \epsilon_2, k_1, l) 4l \cdot k_1}{(l^2 (l-k_1)^2 (l+k_1)^2)} + \frac{4i\epsilon(\epsilon_1, \epsilon_2, k_2, l) 4l \cdot k_2}{(l^2 (l-k_2)^2 (l+k_2)^2)} \Big) J(p,l). \tag{13}$$

where $\epsilon(\epsilon_1, \epsilon_2, k_1, l)$ and $\epsilon(\epsilon_1, \epsilon_2, k_2, l)$ denote the contraction of $\epsilon_1, \epsilon_2, k_1, l$ and $\epsilon_1, \epsilon_2, k_2, l$ with the anti-symmetric tensor ϵ. Assuming that the integral over l' in $J(p,l)$ is finite, the integration over l in the above expression will produce terms proportional to $\epsilon(\epsilon_1, \epsilon_2, k_1, k_1)$, $\epsilon(\epsilon_1, \epsilon_2, k_1, l')\, l' \cdot k_1$ for k_1-term and $\epsilon(\epsilon_1, \epsilon_2, k_2, k_2)$, $\epsilon(\epsilon_1, \epsilon_2, k_2, l')\, l' \cdot k_2$ for k_2-term in Eq. (13). Since $\epsilon(\epsilon_1, \epsilon_2, k_1, k_1) = 0$, $\epsilon(\epsilon_1, \epsilon_2, k_2, k_2) = 0$, only the l' term survives after integration over l. After integration over l', only terms proportional to $p \cdot k_1$ and $p \cdot k_2$ survive, but these are $O(p^2)$ and are suppressed in the soft pion limit, in agreement with the Sutherland-Veltman theorem. Provided that the integrals over l and l' are finite, this result does not depend on the detailed form of the BS wave function and the use of the one-gluon exchange kernel in $J(l,p)$. The $\pi^0 \to \gamma\gamma$ decay is then given by the ABJ anomaly which agrees well with experiment. This implies the absence of the pseudoscalar γ_5 component in the pion BS wave function and the pion thus behaves as a longitudinal axial-vector meson. Also, since the momentum-dependent longitudinal axial-vector meson BS wave function generates only the kinetic term, pion remains massless. The pion mass term has to be generated by chiral symmetry breaking term as the σ-term in the σ model [16].

In conclusion, we have derived the Sutherland-Veltman theorem for the $\pi^0 \to \gamma\gamma$ decay considering the pion as a longitudinal axial-vector meson $q\bar{q}$ bound state. This allows us to say that the pion could be a $q\bar{q}$ bound state at the same time a Nambu-Goldstone boson of chiral symmetry with the two-photon decay given by PCAC and the ABJ chiral anomaly. The momentum-dependent BS wave function could then be used to obtain the derivative couplings with hadrons, in agreement with chiral symmetry.

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[1] S. L. Adler, Phys. Rev. **177**, 2426 (1969).

[2] S. L. Adler, *Lectures on Elementary particles and quantum field theory*, Brandeis University Summer Institute in Theoretical Physics, eds. S. Deser, M. Grisaru and H. Pendleton (MIT Press, Cambridge, MA).

[3] J. S. Bell and R. Jackiw, Nuovo Cimento **60A**, 37 (1969).

[4] C. Haye and N. Isgur, Phys. Rev. D **25**, 1944 (1982).

[5] S. Godfrey and N. Isgur, Phys. Rev. D **32**, 189 (1985).

[6] E. S. Ackleh and T. Barnes, Phys. Rev. D **45**, 232 (1992).

[7] C. R. Münz, J. Resag, B. C. Mestsch, and H. R. Petry Nucl. Phys. A **578**, 418 (1994).

[8] D. G. Sutherland, Nucl. Phys. **B 2**, 433 (1967).

[9] M. Veltman, Proc. R. Soc. 301A, 107 (1967).

[10] See, e.g. T. N. Pham and X. Y. Pham, Phys. Lett. B **247** 438 (1990).

[11] E. E. Salpeter and H. A. Bethe, Phys. Rev. **84**, 1232 (1951).

[12] P. Jain and H. J. Munczek, Phys. Rev. D **44**, 1873 (1991).

[13] T. N. Pham, Phys. Rev. D **19**, 707 (1979).

[14] J. Resag, C. R. Münz, B. C. Mestsch, and H. R. Petry Nucl. Phys. A **578**, 397 (1994).

[15] M. E. Peskin and D. V. Schroeder, *An Introduction to* **QUANTUM FIELD THEORY**, Addison-Wesley Publishing Company, New York (1995).

[16] M. Gell-Mann and M. Levy, Nuovo Cim. **16**, 705 (1960).